32

ELECTROMAGNETIC WAVES

32.1. IDENTIFY: Since the speed is constant, distance $x = ct$.

SET UP: The speed of light is $c = 3.00\times10^8$ m/s. $1\text{ y} = 3.156\times10^7$ s.

EXECUTE: **(a)** $t = \dfrac{x}{c} = \dfrac{3.84\times10^8\text{ m}}{3.00\times10^8\text{ m/s}} = 1.28\text{ s}$

(b) $x = ct = (3.00\times10^8\text{ m/s})(8.61\text{ y})(3.156\times10^7\text{ s/y}) = 8.15\times10^{16}\text{ m} = 8.15\times10^{13}\text{ km}$

EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.

32.2. IDENTIFY: Find the direction of propagation of an electromagnetic wave if we know the directions of the electric and magnetic fields.

SET UP: The direction of propagation of an electromagnetic wave is in the direction of $\vec{E}\times\vec{B}$, which is related to the directions of $\vec{E}$ and $\vec{B}$ according to the right-hand rule for the cross product. The directions of $\vec{E}$ and $\vec{B}$ in each case are shown in Figures 32.2a-d.

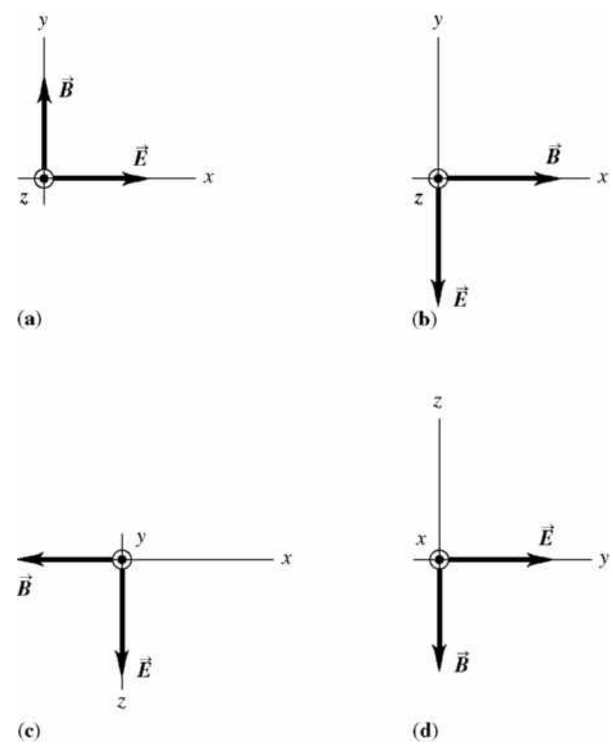


Figure 32.2

EXECUTE: **(a)** The wave is propagating in the $+z$ direction.

(b) $+z$ direction.

(c) $-y$ direction.

(d) $-x$ direction.

EVALUATE: In each case, the direction of propagation is perpendicular to the plane of $\vec{E}$ and $\vec{B}$.

32.3. IDENTIFY: $E_{max} = cB_{max}$. $\vec{E} \times \vec{B}$ is in the direction of propagation.

SET UP: $c = 3.00 \times 10^8$ m/s. $E_{max} = 4.00$ V/m.

EXECUTE: $B_{max} = E_{max}/c = 1.33 \times 10^{-8}$ T. For $\vec{E}$ in the $+x$-direction, $\vec{E} \times \vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

EVALUATE: $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.

32.4. IDENTIFY and SET UP: The direction of propagation is given by $\vec{E} \times \vec{B}$.

EXECUTE: **(a)** $\hat{\boldsymbol{S}} = \hat{\boldsymbol{i}} \times (-\hat{\boldsymbol{j}}) = -\hat{\boldsymbol{k}}$.

(b) $\hat{\boldsymbol{S}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$.

(c) $\hat{\boldsymbol{S}} = (-\hat{\boldsymbol{k}}) \times (-\hat{\boldsymbol{i}}) = \hat{\boldsymbol{j}}$.

(d) $\hat{\boldsymbol{S}} = \hat{\boldsymbol{i}} \times (-\hat{\boldsymbol{k}}) = \hat{\boldsymbol{j}}$.

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.

32.5. IDENTIFY: Knowing the wavelength and speed of x rays, find their frequency, period, and wave number. All electromagnetic waves travel through vacuum at the speed of light.

SET UP: $c = 3.00 \times 10^8$ m/s. $c = f\lambda$. $T = \dfrac{1}{f}$. $k = \dfrac{2\pi}{\lambda}$.

EXECUTE: $f = \dfrac{c}{\lambda} = \dfrac{3.0 \times 10^8 \text{ m/s}}{0.10 \times 10^{-9} \text{ m}} = 3.0 \times 10^{18}$ Hz,

$T = \dfrac{1}{f} = \dfrac{1}{3.0 \times 10^{18} \text{ Hz}} = 3.3 \times 10^{-19}$ s, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.10 \times 10^{-9} \text{ m}} = 6.3 \times 10^{10} \text{ m}^{-1}$.

EVALUATE: The frequency of the x rays is much higher than the frequency of visible light, so their period is much shorter.

32.6. IDENTIFY: $c = f\lambda$ and $k = \dfrac{2\pi}{\lambda}$.

SET UP: $c = 3.00 \times 10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda}$. UVA: 7.50×10^{14} Hz to 9.38×10^{14} Hz. UVB: 9.38×10^{14} Hz to 1.07×10^{15} Hz.

(b) $k = \dfrac{2\pi}{\lambda}$. UVA: 1.57×10^7 rad/m to 1.96×10^7 rad/m. UVB: 1.96×10^7 rad/m to 2.24×10^7 rad/m.

EVALUATE: Larger λ corresponds to smaller f and k.

32.7. IDENTIFY: $c = f\lambda$. $E_{max} = cB_{max}$. $k = 2\pi/\lambda$. $\omega = 2\pi f$.

SET UP: Since the wave is traveling in empty space, its wave speed is $c = 3.00 \times 10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^8 \text{ m/s}}{432 \times 10^{-9} \text{ m}} = 6.94 \times 10^{14}$ Hz

(b) $E_{max} = cB_{max} = (3.00 \times 10^8 \text{ m/s})(1.25 \times 10^{-6} \text{ T}) = 375$ V/m

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{432 \times 10^{-9} \text{ m}} = 1.45 \times 10^7$ rad/m. $\omega = (2\pi \text{ rad})(6.94 \times 10^{14} \text{ Hz}) = 4.36 \times 10^{15}$ rad/s.

$E = E_{max}\cos(kx - \omega t) = (375 \text{ V/m})\cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$

$B = B_{max}\cos(kx - \omega t) = (1.25 \times 10^{-6} \text{ T})\cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$

EVALUATE: The $\cos(kx - \omega t)$ factor is common to both the electric and magnetic field expressions, since these two fields are in phase.

32.8. **IDENTIFY:** $c = f\lambda$. $E_{\max} = cB_{\max}$. Apply Eqs. (32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{435\times10^{-9} \text{ m}} = 6.90\times10^{14}$ Hz

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{2.70\times10^{-3} \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 9.00\times10^{-12}$ T

(c) $k = \dfrac{2\pi}{\lambda} = 1.44\times10^7$ rad/m. $\omega = 2\pi f = 4.34\times10^{15}$ rad/s. If $\vec{E}(z,t) = \hat{\boldsymbol{i}}E_{\max}\cos(kz + \omega t)$, then $\vec{B}(z,t) = -\hat{\boldsymbol{j}}B_{\max}\cos(kz + \omega t)$, so that $\vec{E}\times\vec{B}$ will be in the $-\hat{\boldsymbol{k}}$ direction.

$\vec{E}(z,t) = \hat{\boldsymbol{i}}(2.70\times10^{-3} \text{ V/m})\cos([1.44\times10^7 \text{ rad/m}]z + [4.34\times10^{15} \text{ rad/s}]t)$ and

$\vec{B}(z,t) = -\hat{\boldsymbol{j}}(9.00\times10^{-12} \text{ T})\cos([1.44\times10^7 \text{ rad/m}]z + [4.34\times10^{15} \text{ rad/s}]t)$.

EVALUATE: The directions of $\vec{E}$ and $\vec{B}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz + \omega t$ since the wave is traveling in the $-z$-direction. Waves for visible light have very high frequencies.

32.9. **IDENTIFY:** Electromagnetic waves propagate through air at essentially the speed of light. Therefore, if we know their wavelength, we can calculate their frequency or vice versa.

SET UP: The wave speed is $c = 3.00\times10^8$ m/s. $c = f\lambda$.

EXECUTE: **(a)** (i) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^3 \text{ m}} = 6.0\times10^4$ Hz.

(ii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-6} \text{ m}} = 6.0\times10^{13}$ Hz.

(iii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-9} \text{ m}} = 6.0\times10^{16}$ Hz.

(b) (i) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{6.50\times10^{21} \text{ Hz}} = 4.62\times10^{-14} \text{ m} = 4.62\times10^{-5}$ nm.

(ii) $\lambda = \dfrac{3.00\times10^8 \text{ m/s}}{590\times10^3 \text{ Hz}} = 508 \text{ m} = 5.08\times10^{11}$ nm.

EVALUATE: Electromagnetic waves cover a huge range in frequency and wavelength.

32.10. **IDENTIFY:** For an electromagnetic wave propagating in the negative x direction, $E = E_{\max}\cos(kx + \omega t)$. $\omega = 2\pi f$ and $k = \dfrac{2\pi}{\lambda}$. $T = \dfrac{1}{f}$. $E_{\max} = cB_{\max}$.

SET UP: $E_{\max} = 375$ V/m, $k = 1.99\times10^7$ rad/m and $\omega = 5.97\times10^{15}$ rad/s.

EXECUTE: **(a)** $B_{\max} = \dfrac{E_{\max}}{c} = 1.25\ \mu\text{T}$.

(b) $f = \dfrac{\omega}{2\pi} = 9.50\times10^{14}$ Hz. $\lambda = \dfrac{2\pi}{k} = 3.16\times10^{-7} \text{ m} = 316$ nm. $T = \dfrac{1}{f} = 1.05\times10^{-15}$ s. This wavelength is too short to be visible.

(c) $c = f\lambda = (9.50\times10^{14} \text{ Hz})(3.16\times10^{-7} \text{ m}) = 3.00\times10^8$ m/s. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

EVALUATE: $c = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\left(\dfrac{2\pi}{k}\right) = \dfrac{\omega}{k}$ is an alternative expression for the wave speed.

32.11. **IDENTIFY** and **SET UP:** Compare the $\vec{E}(y,t)$ given in the problem to the general form given by Eq. (32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

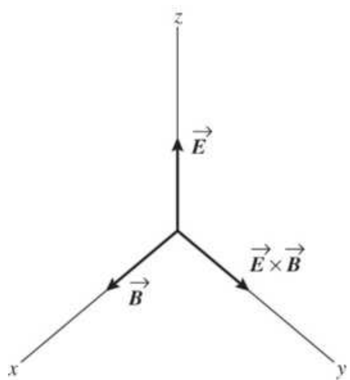
(a) EXECUTE: The equation for the electric field contains the factor $\cos(ky - \omega t)$ so the wave is traveling in the $+y$-direction.

(b) $\vec{E}(y,t) = (3.10\times10^5\ \text{V/m})\hat{k}\cos[ky - (12.65\times10^{12}\ \text{rad/s})t]$

Comparing to Eq. (32.17) gives $\omega = 12.65\times10^{12}\ \text{rad/s}$

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} \text{ so } \lambda = \frac{2\pi c}{\omega} = \frac{2\pi(2.998\times10^8\ \text{m/s})}{(12.65\times10^{12}\ \text{rad/s})} = 1.49\times10^{-4}\ \text{m}$$

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $+z$-direction then $\vec{B}$ must be in the $+x$-direction, as shown in Figure 32.11.

Figure 32.11

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{12.65\times10^{12}\ \text{rad/s}}{2.998\times10^8\ \text{m/s}} = 4.22\times10^4\ \text{rad/m}$$

$E_{\text{max}} = 3.10\times10^5\ \text{V/m}$

Then $B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = \dfrac{3.10\times10^5\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 1.03\times10^{-3}\ \text{T}$

Using Eq. (32.17) and the fact that $\vec{B}$ is in the $+\hat{i}$ direction when $\vec{E}$ is in the $+\hat{k}$ direction,

$\vec{B} = +(1.03\times10^{-3}\ \text{T})\hat{i}\cos[(4.22\times10^4\ \text{rad/m})y - (12.65\times10^{12}\ \text{rad/s})t]$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.12. **IDENTIFY:** Apply Eqs. (32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: $B_y(x,t) = -B_{\text{max}}\cos(kx + \omega t)$.

EXECUTE: **(a)** The phase of the wave is given by $kx + \omega t$, so the wave is traveling in the $-x$ direction.

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c}$. $f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^4\ \text{rad/m})(3.0\times10^8\ \text{m/s})}{2\pi} = 6.59\times10^{11}\,\text{Hz}$.

(c) Since the magnetic field is in the $-y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the –z-direction so that $\vec{E}\times\vec{B}$ will be in the $-x$-direction.

$\vec{E}(x,t) = +cB(x,t)\hat{k} = -cB_{\text{max}}\cos(kx + \omega t)\hat{k}$.

$\vec{E}(x,t) = -(c(8.25\times10^{-9}\ \text{T}))\cos((1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t)\hat{k}$.

$\vec{E}(x,t) = -(2.48\ \text{V/m})\cos((1.38\times10^4\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t)\hat{k}$.

EVALUATE: $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

32.13. **IDENTIFY** and **SET UP:** $c = f\lambda$ allows calculation of λ. $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Eq. (32.18) relates the electric and magnetic field amplitudes.

EXECUTE: **(a)** $c = f\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{830\times10^{3}\ \text{Hz}} = 361\ \text{m}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi\ \text{rad}}{361\ \text{m}} = 0.0174\ \text{rad/m}$

(c) $\omega = 2\pi f = (2\pi)(830\times10^{3}\ \text{Hz}) = 5.22\times10^{6}\ \text{rad/s}$

(d) Eq. (32.18): $E_{\text{max}} = cB_{\text{max}} = (2.998\times10^{8}\ \text{m/s})(4.82\times10^{-11}\ \text{T}) = 0.0144\ \text{V/m}$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio braodcast band. The electric and magnetic fields in the wave are very weak.

32.14. **IDENTIFY:** Apply Eq. (32.21). $E_{\text{max}} = cB_{\text{max}}$. $v = f\lambda$.

SET UP: $K = 3.64$. $K_{\text{m}} = 5.18$

EXECUTE: **(a)** $v = \dfrac{c}{\sqrt{KK_{\text{m}}}} = \dfrac{(3.00\times10^{8}\text{m/s})}{\sqrt{(3.64)(5.18)}} = 6.91\times10^{7}\ \text{m/s}$.

(b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^{7}\,\text{m/s}}{65.0\ \text{Hz}} = 1.06\times10^{6}\ \text{m}$.

(c) $B_{\text{max}} = \dfrac{E_{\text{max}}}{v} = \dfrac{7.20\times10^{-3}\ \text{V/m}}{6.91\times10^{7}\ \text{m/s}} = 1.04\times10^{-10}\ \text{T}$.

EVALUATE: The wave travels slower in this material than in air.

32.15. **IDENTIFY** and **SET UP:** $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use Eq. (32.22) to calculate *n* and *K*.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 3.81\times10^{-7}\ \text{m}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 5.26\times10^{-7}\ \text{m}$

(c) $n = \dfrac{c}{v} = \dfrac{2.998\times10^{8}\ \text{m/s}}{2.17\times10^{8}\ \text{m/s}} = 1.38$

(d) $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ so $K = n^{2} = (1.38)^{2} = 1.90$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so *n* is always greater than unity.

32.16. **IDENTIFY:** We want to find the amount of energy given to each receptor cell and the amplitude of the magnetic field at the cell.

SET UP: Intensity is average power per unit area and power is energy per unit time.
$I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^{2}$, $I = P/A$, and $E_{\text{max}} = cB_{\text{max}}$.

EXECUTE: **(a)** For the beam, the energy is $U = Pt = (2.0\times10^{12}\ \text{W})(4.0\times10^{-9}\ \text{s}) = 8.0\times10^{3}\ \text{J} = 8.0\ \text{kJ}$. This energy is spread uniformly over 100 cells, so the energy given to each cell is 80 J.

(b) The cross-sectional area of each cell is $A = \pi r^{2}$, with $r = 2.5\times10^{-6}$ m.

$$I = \frac{P}{A} = \frac{2.0\times10^{12}\ \text{W}}{(100)\pi(2.5\times10^{-6}\ \text{m})^{2}} = 1.0\times10^{21}\ \text{W/m}^{2}.$$

(c) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^{21}\ \text{W/m}^{2})}{(8.85\times10^{-12}\ \text{C}^{2}/\text{N}\cdot\text{m}^{2})(3.00\times10^{8}\ \text{m/s})}} = 8.7\times10^{11}\ \text{V/m}$.

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 2.9\times10^{3}\ \text{T}.$$

EVALUATE: Both the electric field and magnetic field are very strong compared to ordinary fields.

32.17. **IDENTIFY:** $I = P/A$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2 \cdot \epsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75\ \text{W})}{4\pi(3.0\times10^{-2}\ \text{m})^2} = 330\ \text{W/m}^2$.

(b) $E_{\max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(330\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 500\ \text{V/m}$.

$B_{\max} = \dfrac{E_{\max}}{c} = 1.7\times10^{-6}\ \text{T} = 1.7\ \mu\text{T}$.

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.18. **IDENTIFY:** The intensity of the electromagnetic wave is given by Eq. (32.29): $I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = \epsilon_0 c E_{\text{rms}}^2$.

The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}\ \text{F/m}$

EXECUTE: $\text{Energy} = \epsilon_0 c E_{\text{rms}}^2 At = (8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})(0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\ \mu\text{J}$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.19. **IDENTIFY** and **SET UP:** Use Eq. (32.29) to calculate I, Eq. (32.18) to calculate $B_{\max}$, and use $I = P_{\text{av}}/4\pi r^2$ to calculate P_{av}.

(a) EXECUTE: $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$; $E_{\max} = 0.090$ V/m, so $I = 1.1\times10^{-5}\ \text{W/m}^2$

(b) $E_{\max} = cB_{\max}$ so $B_{\max} = E_{\max}/c = 3.0\times10^{-10}\ \text{T}$

(c) $P_{\text{av}} = I(4\pi r^2) = (1.075\times10^{-5}\ \text{W/m}^2)(4\pi)(2.5\times10^3\ \text{m})^2 = 840\ \text{W}$

(d) EVALUATE: The calculation in part (c) assumes that the transmitter emits uniformly in all directions.

32.20. **IDENTIFY** and **SET UP:** $I = P_{\text{av}}/A$ and $I = \epsilon_0 c E_{\text{rms}}^2$.

EXECUTE: **(a)** The average power from the beam is $P_{\text{av}} = IA = (0.800\ \text{W/m}^2)(3.0\times10^{-4}\ \text{m}^2) = 2.4\times10^{-4}\ \text{W}$.

(b) $E_{\text{rms}} = \sqrt{\dfrac{I}{\epsilon_0 c}} = \sqrt{\dfrac{0.800\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})}} = 17.4\ \text{V/m}$

EVALUATE: The laser emits radiation only in the direction of the beam.

32.21. **IDENTIFY:** $I = P_{\text{av}}/A$

SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.

EXECUTE: $P_{\text{av}} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\ \text{m})^2 = 2.5\times10^{25}\ \text{W}$

EVALUATE: The intensity decreases with distance from the star as $1/r^2$.

32.22. **IDENTIFY** and **SET UP**: $c = f\lambda$, $E_{\max} = cB_{\max}$ and $I = E_{\max}B_{\max}/2\mu_0$

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz}$.

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\ \text{T}$.

(c) $I = S_{\text{av}} = \dfrac{E_{\max}B_{\max}}{2\mu_0} = \dfrac{(0.0540\ \text{V/m})(1.80\times10^{-10}\ \text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2$.

EVALUATE: Alternatively, $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$.

32.23. **IDENTIFY:** $P_{av} = IA$ and $I = E_{max}^2/2\mu_0 c$

SET UP: The surface area of a sphere is $A = 4\pi r^2$.

EXECUTE: $P_{av} = S_{av}A = \left(\dfrac{E_{max}^2}{2c\mu_0}\right)(4\pi r^2)$. $E_{max} = \sqrt{\dfrac{P_{av}c\mu_0}{2\pi r^2}} = \sqrt{\dfrac{(60.0\text{ W})(3.00\times10^8\text{ m/s})\mu_0}{2\pi(5.00\text{ m})^2}} = 12.0\text{ V/m}$.

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{12.0\text{ V/m}}{3.00\times10^8\text{ m/s}} = 4.00\times10^{-8}\text{ T}$.

EVALUATE: E_{max} and B_{max} are both inversely proportional to the distance from the source.

32.24. **IDENTIFY:** The Poynting vector is $\vec{S} = \vec{E}\times\vec{B}$.

SET UP: The electric field is in the $-y$-direction, and the magnetic field is in the $+z$-direction.

$\cos^2\phi = \frac{1}{2}(1+\cos 2\phi)$

EXECUTE: **(a)** $\hat{S} = \hat{E}\times\hat{B} = (-\hat{j})\times\hat{k} = -\hat{i}$. The Poynting vector is in the $-x$-direction, which is the direction of propagation of the wave.

(b) $S(x, t) = \dfrac{E(x, t)B(x, t)}{\mu_0} = \dfrac{E_{max}B_{max}}{\mu_0}\cos^2(kx+\omega t) = \dfrac{E_{max}B_{max}}{2\mu_0}(1+\cos(2(\omega t+kx)))$. But over one period, the cosine function averages to zero, so we have $S_{av} = \dfrac{E_{max}B_{max}}{2\mu_0}$. This is Eq. (32.29).

EVALUATE: We can also show that these two results also apply to the wave represented by Eq. (32.17).

32.25. **IDENTIFY:** Use the radiation pressure to find the intensity, and then $P_{av} = I(4\pi r^2)$.

SET UP: For a perfectly absorbing surface, $p_{rad} = \dfrac{I}{c}$.

EXECUTE: $p_{rad} = I/c$ so $I = cp_{rad} = 2.70\times10^3\text{ W/m}^2$. Then

$P_{av} = I(4\pi r^2) = (2.70\times10^3\text{ W/m}^2)(4\pi)(5.0\text{ m})^2 = 8.5\times10^5\text{ W}$.

EVALUATE: Even though the source is very intense the radiation pressure 5.0 m from the surface is very small.

32.26. **IDENTIFY:** The intensity and the energy density of an electromagnetic wave depends on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{av}/A$, and the average radiation pressure is $P_{av} = 2I/c$, where $I = \frac{1}{2}\epsilon_0 cE_{max}^2$. The energy density is $u = \epsilon_0 E^2$.

EXECUTE: **(a)** $I = P_{av}/A = \dfrac{316{,}000\text{ W}}{2\pi(5000\text{ m})^2} = 0.00201\text{ W/m}^2$. $p_{rad} = 2I/c = \dfrac{2(0.00201\text{ W/m}^2)}{3.00\times10^8\text{ m/s}} = 1.34\times10^{-11}\text{ Pa}$

(b) $I = \frac{1}{2}\epsilon_0 cE_{max}^2$ gives

$$E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00201\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})}} = 1.23\text{ N/C}$$

$$B_{max} = E_{max}/c = (1.23\text{ N/C})/(3.00\times10^8\text{ m/s}) = 4.10\times10^{-9}\text{ T}$$

(c) $u = \epsilon_0 E^2$, so $u_{av} = \epsilon_0(E_{rms})^2$ and $E_{rms} = \dfrac{E_{max}}{\sqrt{2}}$, so

$$u_{av} = \frac{\epsilon_0 E_{max}^2}{2} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.23\text{ N/C})^2}{2} = 6.69\times10^{-12}\text{ J/m}^3$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.27. **IDENTIFY:** We know the greatest intensity that the eye can safely receive.

SET UP: $I = \dfrac{P}{A}$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $P = IA = (1.0\times10^2\ \text{W/m}^2)\pi(0.75\times10^{-3}\ \text{m})^2 = 1.8\times10^{-4}\ \text{W} = 0.18\ \text{mW}$.

(b) $E = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^2\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 274\ \text{V/m}$. $B_{\max} = \dfrac{E_{\max}}{c} = 9.13\times10^{-7}\ \text{T}$.

(c) $P = 0.18\ \text{mW} = 0.18\ \text{mJ/s}$.

(d) $I = (1.0\times10^2\ \text{W/m}^2)\left(\dfrac{1\ \text{m}}{10^2\ \text{cm}}\right)^2 = 0.010\ \text{W/cm}^2$.

EVALUATE: Both the electric and magnetic fields are quite weak compared to normal laboratory fields.

32.28. **IDENTIFY:** Apply Eqs. (32.32) and (32.33). The average momentum density is given by Eq. (32.30), with S replaced by $S_{\text{av}} = I$.

SET UP: $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$

EXECUTE: **(a)** Absorbed light: $p_{\text{rad}} = \dfrac{I}{c} = \dfrac{2500\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}} = 8.33\times10^{-6}\ \text{Pa}$. Then

$$p_{\text{rad}} = \frac{8.33\times10^{-6}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}} = 8.23\times10^{-11}\ \text{atm}.$$

(b) Reflecting light: $p_{\text{rad}} = \dfrac{2I}{c} = \dfrac{2(2500\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}} = 1.67\times10^{-5}\ \text{Pa}$. Then

$$p_{\text{rad}} = \frac{1.67\times10^{-5}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}} = 1.65\times10^{-10}\ \text{atm}.$$

(c) The momentum density is $\dfrac{dp}{dV} = \dfrac{S_{\text{av}}}{c^2} = \dfrac{2500\ \text{W/m}^2}{(3.0\times10^8\ \text{m/s})^2} = 2.78\times10^{-14}\ \text{kg/m}^2\cdot\text{s}$.

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.29. **IDENTIFY:** We know the wavelength and power of the laser beam, as well as the area over which it acts.

SET UP: $P = IA$. $A = \pi r^2$. $E_{\max} = cB_{\max}$. The intensity $I = S_{\text{av}}$ is related to the maximum electric field by $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. The average energy density u_{av} is related to the intensity I by $I = u_{\text{av}} c$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{0.500\times10^{-3}\ \text{W}}{\pi(0.500\times10^{-3}\ \text{m})^2} = 637\ \text{W/m}^2$.

(b) $E_{\max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(637\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 693\ \text{V/m}$. $B_{\max} = \dfrac{E_{\max}}{c} = 2.31\ \mu\text{T}$.

(c) $u_{\text{av}} = \dfrac{I}{c} = \dfrac{637\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = 2.12\times10^{-6}\ \text{J/m}^3$.

EVALUATE: The fields are very weak, so a cubic meter of space contains only about $2\ \mu\text{J}$ of energy.

32.30. **IDENTIFY:** We know the intensity of the solar light and the area over which it acts. We can use the light intensity to find the force the light exerts on the sail, and then use the sail's density to find its mass. Newton's second law will then give the acceleration of the sail.

SET UP: For a reflecting surface the pressure is $\dfrac{2I}{c}$. Pressure is force per unit area, and $F_{\text{net}} = ma$. The mass of the sail is its volume V times its density ρ. The area of the sail is πr^2, with $r = 4.5$ m. Its volume is $\pi r^2 t$, where $t = 7.5\times10^{-6}$ m is its thickness.

EXECUTE: **(a)** $F=\left(\dfrac{2I}{c}\right)A=\dfrac{2(1400\text{ W/m}^2)}{3.00\times10^8\text{ m/s}}\pi(4.5\text{ m})^2=5.9\times10^{-4}\text{ N}.$

(b) $m=\rho V=(1.74\times10^3\text{ kg/m}^3)\pi(4.5\text{ m})^2(7.5\times10^{-6}\text{ m})=0.83\text{ kg}.$

$a=\dfrac{F}{m}=\dfrac{5.9\times10^{-4}\text{ N}}{0.83\text{ kg}}=7.1\times10^{-4}\text{ m/s}^2.$

(c) With this acceleration it would take the sail $1.4\times10^6\text{ s}=16\text{ days}$ to reach a speed of 1 km/s. This would be useful only in specialized applications. The acceleration could be increased by decreasing the mass of the sail, either by reducing its density or its thickness.

EVALUATE: The calculation assumed the only force on the sail is that due to the radiation pressure. The sun would also exert a gravitational force on the sail, which could be significant.

32.31. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $\left(2\frac{1}{2}\right)\lambda=L,$ and the frequency of these waves obeys the equation $f\lambda=c.$

EXECUTE: **(a)** Since $\left(2\frac{1}{2}\right)\lambda=L,$ we have $L=(5/2)(12.2\text{ cm})=30.5\text{ cm}.$

(b) Solving for the frequency gives $f=c/\lambda=(3.00\times10^8\text{ m/s})/(0.122\text{ m})=2.46\times10^9\text{ Hz}.$

(c) $L=35.5\text{ cm}$ in this case. $\left(2\frac{1}{2}\right)\lambda=L,$ so $\lambda=2L/5=2(35.5\text{ cm})/5=14.2\text{ cm}.$

$$f=c/\lambda=(3.00\times10^8\text{ m/s})/(0.142\text{ m})=2.11\times10^9\text{ Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.32. **IDENTIFY:** The electric field at the nodes is zero, so there is no force on a point charge placed at a node.

SET UP: The location of the nodes is given by Eq. (32.36), where x is the distance from one of the planes. $\lambda=c/f.$

EXECUTE: $\Delta x_{\text{nodes}}=\dfrac{\lambda}{2}=\dfrac{c}{2f}=\dfrac{3.00\times10^8\text{ m/s}}{2(7.50\times10^8\text{ Hz})}=0.200\text{ m}=20.0\text{ cm}.$ There must be nodes at the planes, which are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm from one plane that a point charge will remain at rest, since the electric fields there are zero.

EVALUATE: The magnetic field amplitude at these points isn't zero, but the magnetic field doesn't exert a force on a stationary charge.

32.33. **IDENTIFY** and **SET UP:** Apply Eqs. (32.36) and (32.37).

EXECUTE: **(a)** By Eq. (32.37) we see that the nodal planes of the $\vec{B}$ field are a distance $\lambda/2$ apart, so $\lambda/2=3.55\text{ mm}$ and $\lambda=7.10\text{ mm}.$

(b) By Eq. (32.36) we see that the nodal planes of the $\vec{E}$ field are also a distance $\lambda/2=3.55\text{ mm}$ apart.

(c) $v=f\lambda=(2.20\times10^{10}\text{ Hz})(7.10\times10^{-3}\text{ m})=1.56\times10^8\text{ m/s}.$

EVALUATE: The spacing between the nodes of $\vec{E}$ is the same as the spacing between the nodes of $\vec{B}$. Note that $v<c,$ as it must.

32.34. **IDENTIFY:** The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs. (32.26) and (32.27).

SET UP: $\lambda=\dfrac{c}{f}=\dfrac{3.00\times10^8\text{ m/s}}{75.0\times10^6\text{ Hz}}=4.00\text{ m}$

EXECUTE: **(a)** $\Delta x=\dfrac{\lambda}{2}=2.00\text{ m}.$

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is $\dfrac{\lambda}{4}=\dfrac{\Delta x}{2}=\dfrac{2.00\text{ m}}{2}=1.00\text{ m}.$

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.35. **(a) IDENTIFY** and **SET UP:** The distance between adjacent nodal planes of $\vec{B}$ is $\lambda/2$. There is an antinodal plane of $\vec{B}$ midway between any two adjacent nodal planes, so the distance between a nodal plane and an adjacent antinodal plane is $\lambda/4$. Use $v = f\lambda$ to calculate λ.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{2.10\times10^{8}\ \text{m/s}}{1.20\times10^{10}\ \text{Hz}} = 0.0175\ \text{m}$

$\dfrac{\lambda}{4} = \dfrac{0.0175\ \text{m}}{4} = 4.38\times10^{-3}\ \text{m} = 4.38\ \text{mm}$

(b) IDENTIFY and **SET UP:** The nodal planes of $\vec{E}$ are at $x = 0,\ \lambda/2, \lambda, 3\lambda/2, \ldots,$ so the antinodal planes of $\vec{E}$ are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots.$ The nodal planes of $\vec{B}$ are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots,$ so the antinodal planes of $\vec{B}$ are at $\lambda/2, \lambda, 3\lambda/2, \ldots.$

EXECUTE: The distance between adjacent antinodal planes of $\vec{E}$ and antinodal planes of $\vec{B}$ is therefore $\lambda/4 = 4.38$ mm.

(c) From Eqs. (32.36) and (32.37) the distance between adjacent nodal planes of $\vec{E}$ and $\vec{B}$ is $\lambda/4 = 4.38$ mm.

EVALUATE: The nodes of $\vec{E}$ coincide with the antinodes of $\vec{B}$ and conversely. The nodes of $\vec{B}$ and the nodes of $\vec{E}$ are equally spaced.

32.36. **IDENTIFY:** Evaluate the derivatives of the expressions for $E_y(x,t)$ and $B_z(x,t)$ that are given in Eqs. (32.34) and (32.35).

SET UP: $\dfrac{\partial}{\partial x}\sin kx = k\cos kx,\ \dfrac{\partial}{\partial t}\sin\omega t = \omega\cos\omega t.\ \dfrac{\partial}{\partial x}\cos kx = -k\sin kx,\ \dfrac{\partial}{\partial t}\cos\omega t = -\omega\sin\omega t.$

EXECUTE: **(a)** $\dfrac{\partial^2 E_y(x,t)}{\partial x^2} = \dfrac{\partial^2}{\partial x^2}(-2E_{\text{max}}\sin kx\sin\omega t) = \dfrac{\partial}{\partial x}(-2kE_{\text{max}}\cos kx\sin\omega t)$ and

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = 2k^2E_{\text{max}}\sin kx\sin\omega t = \frac{\omega^2}{c^2}2E_{\text{max}}\sin kx\sin\omega t = \epsilon_0\mu_0\frac{\partial^2 E_y(x,t)}{\partial t^2}.$$

Similarly: $\dfrac{\partial^2 B_z(x,t)}{\partial x^2} = \dfrac{\partial^2}{\partial x^2}(-2B_{\text{max}}\cos kx\cos\omega t) = \dfrac{\partial}{\partial x}(+2kB_{\text{max}}\sin kx\cos\omega t)$ and

$$\frac{\partial^2 B_z(x,t)}{\partial x^2} = 2k^2B_{\text{max}}\cos kx\cos\omega t = \frac{\omega^2}{c^2}2B_{\text{max}}\cos kx\cos\omega t = \epsilon_0\mu_0\frac{\partial^2 B_z(x,t)}{\partial t^2}.$$

(b) $\dfrac{\partial E_y(x,t)}{\partial x} = \dfrac{\partial}{\partial x}(-2E_{\text{max}}\sin kx\sin\omega t) = -2kE_{\text{max}}\cos kx\sin\omega t.$

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\omega}{c}2E_{\text{max}}\cos kx\sin\omega t = -\omega 2\frac{E_{\text{max}}}{c}\cos kx\sin\omega t = -\omega 2B_{\text{max}}\cos kx\sin\omega t.$$

$$\frac{\partial E_y(x,t)}{\partial x} = +\frac{\partial}{\partial t}(2B_{\text{max}}\cos kx\cos\omega t) = -\frac{\partial B_z(x,t)}{\partial t}.$$

Similarly: $-\dfrac{\partial B_z(x,t)}{\partial x} = \dfrac{\partial}{\partial x}(+2B_{\text{max}}\cos kx\cos\omega t) = -2kB_{\text{max}}\sin kx\cos\omega t.$

$$-\frac{\partial B_z(x,t)}{\partial x} = -\frac{\omega}{c}2B_{\text{max}}\sin kx\cos\omega t = -\frac{\omega}{c^2}2cB_{\text{max}}\sin kx\cos\omega t.$$

$$-\frac{\partial B_z(x,t)}{\partial x} = -\epsilon_0\mu_0\omega 2E_{\text{max}}\sin kx\cos\omega t = \epsilon_0\mu_0\frac{\partial}{\partial t}(-2E_{\text{max}}\sin kx\sin\omega t) = \epsilon_0\mu_0\frac{\partial E_y(x,t)}{\partial t}.$$

EVALUATE: The standing waves are linear superpositions of two traveling waves of the same k and ω.

32.37. IDENTIFY: We know the wavelength and power of a laser beam as well as the area over which it acts and the duration of a pulse.

SET UP: The energy is $U = Pt$. For absorption the radiation pressure is $\frac{I}{c}$, where $I = \frac{P}{A}$. The wavelength in the eye is $\lambda = \frac{\lambda_0}{n}$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$ and $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $U = Pt = (250\times10^{-3}\ \text{W})(1.50\times10^{-3}\ \text{s}) = 3.75\times10^{-4}\ \text{J} = 0.375\ \text{mJ}$.

(b) $I = \frac{P}{A} = \frac{250\times10^{-3}\ \text{W}}{\pi(255\times10^{-6}\ \text{m})^2} = 1.22\times10^6\ \text{W/m}^2$. The average pressure is

$\frac{I}{c} = \frac{1.22\times10^6\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = 4.08\times10^{-3}\ \text{Pa}$.

(c) $\lambda = \frac{\lambda_0}{n} = \frac{810\ \text{nm}}{1.34} = 604\ \text{nm}$. $f = \frac{v}{\lambda} = \frac{c}{\lambda_0} = \frac{3.00\times10^8\ \text{m/s}}{810\times10^{-9}\ \text{m}} = 3.70\times10^{14}\ \text{Hz}$; f is the same in the air and in the vitreous humor.

(d) $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(1.22\times10^6\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 3.03\times10^4\ \text{V/m}$.

$B_{\max} = \frac{E_{\max}}{c} = 1.01\times10^{-4}\ \text{T}$.

EVALUATE: The intensity of the beam is high, as it must be to weld tissue, but the pressure it exerts on the retina is only around 10^{-8} that of atmospheric pressure. The magnetic field in the beam is about twice that of the earth's magnetic field.

32.38. IDENTIFY: Evaluate the partial derivatives of the expressions for $E_y(x,t)$ and $B_z(x,t)$.

SET UP: $\frac{\partial}{\partial x}\cos(kx-\omega t) = -k\sin(kx-\omega t)$, $\frac{\partial}{\partial t}\cos(kx-\omega t) = \omega\sin(kx-\omega t)\cdot$

$\frac{\partial}{\partial x}\sin(kx-\omega t) = k\cos(kx-\omega t)$, $\frac{\partial}{\partial t}\sin(kx-\omega t) = -\omega\cos(kx-\omega t)$

EXECUTE: Assume $\vec{E} = E_{\max}\hat{j}\cos(kx-\omega t)$ and $\vec{B} = B_{\max}\hat{k}\cos(kx-\omega t+\phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$. This gives $kE_{\max}\sin(kx-\omega t) = +\omega B_{\max}\sin(kx-\omega t+\phi)$, so $\phi = 0$, and $kE_{\max} = \omega B_{\max}$, so $E_{\max} = \frac{\omega}{k}B_{\max} = \frac{2\pi f}{2\pi/\lambda}B_{\max} = f\lambda B_{\max} = cB_{\max}$. Similarly for Eq. (32.14), $-\frac{\partial B_z}{\partial x} = \epsilon_0\mu_0\frac{\partial E_y}{\partial t}$ gives $kB_{\max}\sin(kx-\omega t+\phi) = \epsilon_0\mu_0\omega E_{\max}\sin(kx-\omega t)$, so $\phi = 0$ and $kB_{\max} = \epsilon_0\mu_0\omega E_{\max}$, so

$B_{\max} = \frac{\epsilon_0\mu_0\omega}{k}E_{\max} = \frac{2\pi f}{c^2 2\pi/\lambda}E_{\max} = \frac{f\lambda}{c^2}E_{\max} = \frac{1}{c}E_{\max}$.

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.39. IDENTIFY: The light exerts pressure on the paper, which produces an upward force. This force must balance the weight of the paper.

SET UP: The weight of the paper is mg. For a totally absorbing surface the radiation pressure is $\frac{I}{c}$ and for a totally reflecting surface it is $\frac{2I}{c}$. The force is $F = PA$, and the intensity is $I = \frac{P}{A}$.

EXECUTE: **(a)** The radiation force must equal the weight of the paper, so $\left(\frac{I}{c}\right)A = mg$.

$I = \frac{mgc}{A} = \frac{(1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(3.00\times10^8\ \text{m/s})}{(0.220\ \text{m})(0.280\ \text{m})} = 7.16\times10^7\ \text{W/m}^2$.

(b) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. Solving for $E_{\max}$ gives

$$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(7.16\times10^7\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.32\times10^5\ \text{V/m}.$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{2.32\times10^5\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 7.74\times10^{-4}\ \text{T}.$$

(c) The pressure is $\frac{2I}{c}$, so $\left(\frac{2I}{c}\right)A = mg$. $I = \frac{mgc}{2A} = 3.58\times10^7\ \text{W/m}^2$.

(d) $I = \frac{P}{A} = \frac{0.500\times10^{-3}\ \text{W}}{\pi(0.500\times10^{-3}\ \text{m})^2} = 637\ \text{W/m}^2$.

EVALUATE: The intensity of this laser is much less than what is needed to support a sheet of paper. And to support the paper, not only must the intensity be large, it also must be over a large area.

32.40. **IDENTIFY:** The average energy density in the electric field is $u_{E,\text{av}} = \frac{1}{2}\epsilon_0 (E^2)_{\text{av}}$ and the average energy density in the magnetic field is $u_{B,\text{av}} = \frac{1}{2\mu_0}(B^2)_{\text{av}}$.

SET UP: $(\cos^2(kx-\omega t))_{\text{av}} = \frac{1}{2}$.

EXECUTE: $E_y(x,t) = E_{\max}\cos(kx-\omega t)$. $u_E = \frac{1}{2}\epsilon_0 E_y^2 = \frac{1}{2}\epsilon_0 E_{\max}^2 \cos^2(kx-\omega t)$ and $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2$.

$B_z(x,t) = B_{\max}\cos(kx-\omega t)$, so $u_B = \frac{1}{2\mu_0}B_z^2 = \frac{1}{2\mu_0}B_{\max}^2\cos^2(kx-\omega t)$ and $u_{B,\text{av}} = \frac{1}{4\mu_0}B_{\max}^2$.

$E_{\max} = cB_{\max}$, so $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 c^2 B_{\max}^2$. $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, so $u_{E,\text{av}} = \frac{1}{2\mu_0}B_{\max}^2$, which equals $u_{B,\text{av}}$.

EVALUATE: Our result allows us to write $u_{\text{av}} = 2u_{E,\text{av}} = \frac{1}{2}\epsilon_0 E_{\max}^2$ and $u_{\text{av}} = 2u_{B,\text{av}} = \frac{1}{2\mu_0}B_{\max}^2$.

32.41. **IDENTIFY:** The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{\text{av}}/A = \frac{1}{2}\epsilon_0 c E_{\max}^2$, and the force is $F = p_{\text{rad}}A$ where $p_{\text{rad}} = I/c$.

EXECUTE: **(a)** $I = P_{\text{av}}/A = (25{,}000\ \text{W})/[4\pi(575\ \text{m})^2] = 0.00602\ \text{W/m}^2$

(b) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$, so $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00602\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.13\ \text{N/C}$.

$$B_{\max} = E_{\max}/c = (2.13\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 7.10\times10^{-9}\ \text{T}$$

(c) $F = p_{\text{rad}}A = (I/c)A = (0.00602\ \text{W/m}^2)(0.150\ \text{m})(0.400\ \text{m})/(3.00\times10^8\ \text{m/s}) = 1.20\times10^{-12}\ \text{N}$

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.42. **IDENTIFY:** $c = f\lambda$. $E_{\max} = cB_{\max}$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. For a totally absorbing surface the radiation pressure is $\frac{I}{c}$.

SET UP: The wave speed in air is $c = 3.00\times10^8\ \text{m/s}$.

EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{3.84\times10^{-2}\ \text{m}} = 7.81\times10^9\ \text{Hz}$

(b) $B_{\max} = \frac{E_{\max}}{c} = \frac{1.35\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 4.50\times10^{-9}\ \text{T}$

(c) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = \frac{1}{2}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})(1.35\ \text{V/m})^2 = 2.42\times10^{-3}\ \text{W/m}^2$

(d) $F = (\text{pressure})A = \dfrac{IA}{c} = \dfrac{(2.42\times10^{-3}\ \text{W/m}^2)(0.240\ \text{m}^2)}{3.00\times10^8\ \text{m/s}} = 1.94\times10^{-12}\ \text{N}$

EVALUATE: The intensity depends only on the amplitudes of the electric and magnetic fields and is independent of the wavelength of the light.

32.43. **(a) IDENTIFY and SET UP:** Calculate I and then use Eq. (32.29) to calculate E_{max} and Eq. (32.18) to calculate B_{max}.

EXECUTE: The intensity is power per unit area: $I = \dfrac{P}{A} = \dfrac{4.60\times10^{-3}\ \text{W}}{\pi(1.25\times10^{-3}\ \text{m})^2} = 937\ \text{W/m}^2.$

$I = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$, so $E_{\text{max}} = \sqrt{2\mu_0 cI}$. $E_{\text{max}} = \sqrt{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.998\times10^8\ \text{m/s})(937\ \text{W/m}^2)} = 840\ \text{V/m}.$

$B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = \dfrac{840\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 2.80\times10^{-6}\ \text{T}.$

EVALUATE: The magnetic field amplitude is quite small compared to laboratory fields.

(b) IDENTIFY and SET UP: Eqs. (24.11) and (30.10) give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\text{max}}\cos(kx-\omega t)$ and the average value of $\cos^2(kx-\omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\text{max}}^2 = \frac{1}{4}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(840\ \text{V/m})^2 = 1.56\times10^{-6}\ \text{J/m}^3$. The energy density in the magnetic field is $u_B = \dfrac{B^2}{2\mu_0}$. The average value is $u_{B,\text{av}} = \dfrac{B_{\text{max}}^2}{4\mu_0} = \dfrac{(2.80\times10^{-6}\ \text{T})^2}{4(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 1.56\times10^{-6}\ \text{J/m}^3.$

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and SET UP: The total energy in this length of beam is the average energy density $(u_{\text{av}} = u_{E,\text{av}} + u_{B,\text{av}} = 3.12\times10^{-6}\ \text{J/m}^3)$ times the volume of this part of the beam.

EXECUTE: $U = u_{\text{av}}LA = (3.12\times10^{-6}\ \text{J/m}^3)(1.00\ \text{m})\pi(1.25\times10^{-3}\ \text{m})^2 = 1.53\times10^{-11}\ \text{J}.$

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00$ m: $U = P\left(\dfrac{L}{c}\right) = (4.60\times10^{-3}\ \text{W})\left(\dfrac{1.00\ \text{m}}{2.998\times10^8\ \text{m/s}}\right) = 1.53\times10^{-11}\ \text{J}$, which checks.

32.44. **IDENTIFY:** We know the electric field in the plastic.

SET UP: The general wave function for the electric field is $E = E_{\text{max}}\cos(kx-\omega t)$. $f = \dfrac{\omega}{2\pi}$, $\lambda = \dfrac{2\pi}{k}$, $v = f\lambda$ and $v = \dfrac{c}{n}$.

EXECUTE: **(a)** By comparing the equation for E to the general form, we have $\omega = 3.02\times10^{15}\ \text{rad/s}$ and $k = 1.39\times10^7\ \text{rad/m}$. $f = \dfrac{\omega}{2\pi} = 4.81\times10^{14}\ \text{Hz}$. $\lambda = \dfrac{2\pi}{k} = 4.52\times10^{-7}\ \text{m} = 452\ \text{nm}$.

$v = f\lambda = 2.17\times10^8\ \text{m/s}.$

(b) $n = \dfrac{c}{v} = \dfrac{3.00\times10^8\ \text{m/s}}{2.17\times10^8\ \text{m/s}} = 1.38.$

(c) In air, $\omega = 3.02\times10^{15}\ \text{rad/s}$, the same as in the plastic. $\lambda_0 = \lambda n = (4.52\times10^{-7}\ \text{m})(1.38) = 6.24\times10^{-7}\ \text{m}$, so $k = \dfrac{2\pi}{\lambda} = 1.01\times10^7\ \text{rad/m}$. The equation for E in air is

$E = (535\ \text{V/m})\cos\left[(1.01\times10^7\ \text{rad/m})x - (3.02\times10^{15}\ \text{rad/s})t\right].$

EVALUATE: In the plastic, k and λ are different from their values in air, but f and ω are the same in both media.

32.45. **IDENTIFY:** $I = P_{av}/A.$ For an absorbing surface, the radiation pressure is $p_{rad} = \frac{I}{c}.$

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{av}/(4\pi r^2).$

EXECUTE: **(a)** At the sun's surface: $I = \frac{P_{av}}{4\pi R^2} = \frac{3.9\times10^{26}\text{ W}}{4\pi(6.96\times10^8\text{ m})^2} = 6.4\times10^7\text{ W/m}^2$ and

$p_{rad} = \frac{I}{c} = \frac{6.4\times10^7\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = 0.21\text{ Pa}.$

Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6\times10^8\text{ W/m}^2$ and $p_{rad} = 0.85\text{ Pa}.$ At the top of the earth's atmosphere, the measured sunlight intensity is 1400 W/m^2 and $p_{rad} = 5\times10^{-6}\text{ Pa},$ which is about 100,000 times less than the values above.

EVALUATE: **(b)** The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.46. **IDENTIFY:** The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = S_{av} = \frac{1}{2}\epsilon_0 c E_{max}^2.$

EXECUTE: Since $I = \frac{1}{2}\epsilon_0 c E_{max}^2,$ $E_{max} \propto \sqrt{I}.$ A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200\text{ m})/(10.0\text{ m}) = 1/50,$ we have $I_{0.20} = 50^2 I_{10}.$ Since $E_{max} \propto \sqrt{I},$ we have $E_{0.20} = 50E_{10} = (50)(1.50\text{ N/C}) = 75.0\text{ N/C}.$

EVALUATE: While the intensity increases by a factor of $50^2 = 2500,$ the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.47. **IDENTIFY:** The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = p_{rad}A = (I/c)A,$ while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2}\epsilon_0 c E_{max}^2.$ Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{net} = I\alpha,$ to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{Abs} = p_{rad}A = (I/c)A = \frac{\frac{1}{2}\epsilon_0 c E_{max}^2 A}{c} = \frac{1}{2}\epsilon_0 A E_{max}^2.$

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 c E_{max}^2.$ The net torque is therefore $\tau_{net} = F_{Refl}(L/2) - F_{Abs}(L/2) = \epsilon_0 A E_{max}^2 L/4.$

Newton's second law for rotation gives $\tau_{net} = I\alpha.$ $\epsilon_0 A E_{max}^2 L/4 = 2m(L/2)^2\alpha.$

Solving for α gives

$$\alpha = \epsilon_0 A E_{max}^2/(2mL) = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.0150\text{ m})^2(1.25\text{ N/C})^2}{(2)(0.00400\text{ kg})(1.00\text{ m})} = 3.89\times10^{-13}\text{ rad/s}^2.$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.48. **IDENTIFY:** The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop.

SET UP: $\Phi_B = B\pi r^2 = \pi r^2 B_{\text{max}} \cos(kx - \omega t)$, taking x for the direction of propagation of the wave. Faraday's law says $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The intensity of the wave is $I = \dfrac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0} B_{\text{max}}^2$, and $f = \dfrac{c}{\lambda}$.

EXECUTE: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = \omega B_{\text{max}} \sin(kx - \omega t)\pi r^2$. $|\mathcal{E}|_{\text{max}} = 2\pi f B_{\text{max}} \pi r^2$.

$f = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^8\ \text{m/s}}{6.90\ \text{m}} = 4.348\times 10^7$ Hz. Solving $I = \dfrac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0} B_{\text{max}}^2$ for B_{max} gives

$$B_{\text{max}} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(0.0195\ \text{W/m}^2)}{3.00\times 10^8\ \text{m/s}}} = 1.278\times 10^{-8}\ \text{T}.$$

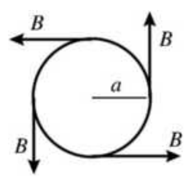
$$|\mathcal{E}|_{\text{max}} = 2\pi(4.348\times 10^7\ \text{Hz})(1.278\times 10^{-8}\ \text{T})\pi(0.075\ \text{m})^2 = 6.17\times 10^{-2}\ \text{V} = 61.7\ \text{mV}.$$

EVALUATE: This voltage is quite small compared to everyday voltages, so it normally would not be noticed. But in very delicate laboratory work, it could be large enough to take into consideration.

32.49. **IDENTIFY** and **SET UP:** In the wire the electric field is related to the current density by Eq. (25.7). Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by Eq. (32.28) and the equation that follows it relates the energy flow through a surface to $\vec{S}$.

EXECUTE: **(a)** The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. From Eq. (25.7), $E = \rho J = \rho I/\pi a^2$. ($E$ is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.49a; take the current to be coming out of the page.



Apply Ampere's law to a circle of radius a.

$\oint \vec{B}\cdot d\vec{l} = B(2\pi a)$

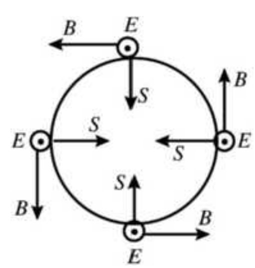
$I_{\text{encl}} = I$

Figure 32.49a

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi a) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi a}$

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.49b.



The direction of $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$ is radially inward.

$$S = \frac{1}{\mu_0}EB = \frac{1}{\mu_0}\left(\frac{\rho I}{\pi a^2}\right)\left(\frac{\mu_0 I}{2\pi a}\right)$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}$$

Figure 32.49b

(d) EVALUATE: Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \dfrac{\rho I^2}{2\pi^2 a^3}(2\pi al) = \dfrac{\rho l I^2}{\pi a^2}$. But $R = \dfrac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.50. **IDENTIFY:** The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4,\ 3\lambda/4,\ 5\lambda/4,\ \ldots,$ which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (3.000\times10^8\ \text{m/s})/(110.0\times10^6\ \text{Hz}) = 2.727\ \text{m}$. So the nodal planes are at $(2.727\ \text{m})/2 = 1.364\ \text{m}$ apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.727\ \text{m})/2 = 10.91\ \text{m}$.

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.51. **IDENTIFY and SET UP:** Find the force on you due to the momentum carried off by the light. Express this force in terms of the radiated power of the flashlight. Use this force to calculate your acceleration and use a constant acceleration equation to find the time.

(a) EXECUTE: $p_{\text{rad}} = I/c$ and $F = p_{\text{rad}}A$ gives $F = IA/c = P_{\text{av}}/c$

$a_x = F/m = P_{\text{av}}/(mc) = (200\ \text{W})/[(150\ \text{kg})(3.00\times10^8\ \text{m/s})] = 4.44\times10^{-9}\ \text{m/s}^2$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$t = \sqrt{2(x-x_0)/a_x} = \sqrt{2(16.0\ \text{m})/(4.44\times10^{-9}\ \text{m/s}^2)} = 8.49\times10^4\ \text{s} = 23.6\ \text{h}$$

EVALUATE: The radiation force is very small. In the calculation we have ignored any other forces on you.

(b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.52. **IDENTIFY:** $P_{\text{av}} = IA$ and $I = \frac{1}{2}\epsilon_0 c E_{\text{max}}^2$. $E_{\text{max}} = cB_{\text{max}}$

SET UP: The power carried by the current i is $P = Vi$.

EXECUTE: $I = \dfrac{P_{\text{av}}}{A} = \frac{1}{2}\epsilon_0 c E_{\text{max}}^2$ and

$$E_{\text{max}} = \sqrt{\frac{2P_{\text{av}}}{A\epsilon_0 c}} = \sqrt{\frac{2Vi}{A\epsilon_0 c}} = \sqrt{\frac{2(5.00\times10^5\ \text{V})(1000\ \text{A})}{(100\ \text{m}^2)\epsilon_0(3.00\times10^8\ \text{m/s})}} = 6.14\times10^4\ \text{V/m}.$$

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{6.14\times10^4\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 2.05\times10^{-4}\ \text{T}.$$

EVALUATE: $I = Vi/A = \dfrac{(5.00\times10^5\ \text{V})(1000\ \text{A})}{100\ \text{m}^2} = 5.00\times10^6\ \text{W/m}^2$. This is a very intense beam spread over a large area.

32.53. **IDENTIFY:** The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/r = GmM/r^2$, where M is the mass of the earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\text{max}}{}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: (a) The period of the orbit is 12 hr. Applying Newton's second law to the satellite gives $mv^2/r = GmM/r^2$, which gives $\dfrac{m(2\pi r/T)^2}{r} = \dfrac{GmM}{r^2}$. Solving for r, we get

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(12\times3600\ \text{s})^2}{4\pi^2}\right]^{1/3} = 2.66\times10^7\ \text{m}$$

The height above the surface is $h = 2.66\times10^7\ \text{m} - 6.38\times10^6\ \text{m} = 2.02\times10^7\ \text{m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0\ \text{W})/[2\pi(2.02\times10^7\ \text{m})^2] = 9.75\times10^{-15}\ \text{W/m}^2$$

(b) $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\text{max}}^2$, so $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(9.75\times10^{-15}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.71\times10^{-6}\ \text{N/C}$

$$B_{\text{max}} = E_{\text{max}}/c = (2.71\times10^{-6}\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 9.03\times10^{-15}\ \text{T}$$

$$t = d/c = (2.02\times10^7\ \text{m})/(3.00\times10^8\ \text{m/s}) = 0.0673\ \text{s}$$

(c) $p_{\text{rad}} = I/\text{c} = (9.75\times10^{-15}\ \text{W/m}^2)/(3.00\times10^8\ \text{m/s}) = 3.25\times10^{-23}\ \text{Pa}$

(d) $\lambda = c/f = (3.00\times10^8\ \text{m/s})/(1575.42\times10^6\ \text{Hz}) = 0.190\ \text{m}$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.54. IDENTIFY: For a totally reflective surface the radiation pressure is $\dfrac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_{\text{g}} = G\dfrac{mM_{\text{sun}}}{r^2}$.

SET UP: The mass of the sun is $M_{\text{sun}} = 1.99\times10^{30}\ \text{kg}$. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: (a) The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{\text{rad}} = \left(\dfrac{2I}{c}\right)A$, where A is the area of the sail. $I = \dfrac{P}{4\pi r^2}$, where r is the distance of the sail from the sun. $F_{\text{rad}} = \left(\dfrac{2A}{c}\right)\left(\dfrac{P}{4\pi r^2}\right) = \dfrac{PA}{2\pi r^2 c}\cdot F_{\text{rad}} = F_{\text{g}}$ so $\dfrac{PA}{2\pi r^2 c} = G\dfrac{mM_{\text{sun}}}{r^2}$.

$$A = \frac{2\pi cGmM_{\text{sun}}}{P} = \frac{2\pi(3.00\times10^8\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$A = 6.42\times10^6\ \text{m}^2 = 6.42\ \text{km}^2$.

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_{\text{g}}$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.55. IDENTIFY and SET UP: The gravitational force is given by Eq. (13.2). Express the mass of the particle in terms of its density and volume. The radiation pressure is given by Eq. (32.32); relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross-sectional area of the particle.

EXECUTE: (a) The gravitational force is $F_{\text{g}} = G\dfrac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho\frac{4}{3}\pi R^3$.

Thus $F_{\text{g}} = \dfrac{4\rho G\pi MR^3}{3r^2}$.

(b) For a totally absorbing surface $p_{\text{rad}} = \dfrac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \dfrac{L}{4\pi r^2}$. Thus $p_{\text{rad}} = \dfrac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{\text{rad}} = p_{\text{rad}} A_{\perp}$, where $A_{\perp} = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus

$$F_{\text{rad}} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}.$$

(c) $F_g = F_{\text{rad}}$

$$\frac{4\rho G\pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}$$

$$R = \frac{3(3.9\times10^{26}\ \text{W})}{16(2.998\times10^8\ \text{m/s})(3000\ \text{kg/m}^3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\pi(1.99\times10^{30}\ \text{kg})}$$

$R = 1.9\times10^{-7}\ \text{m} = 0.19\ \mu\text{m}.$

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\dfrac{F_{\text{rad}}}{F_{\text{g}}} = \left(\dfrac{LR^2}{4cr^2}\right)\left(\dfrac{3r^2}{4\rho G\pi m R^3}\right) = \dfrac{3L}{16c\rho G\pi MR}$. F_{rad} is proportional to R^2 and F_{g} is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{\text{rad}} > F_{\text{g}}$ and the radiation force will drive the particles out of the solar system.

32.56. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E_y(x,t) = E_{\max} e^{-k_{\text{C}}x}\cos(k_{\text{C}}x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_{\text{C}})e^{-k_{\text{C}}x}\cos(k_{\text{C}}x - \omega t) + E_{\max}(-k_{\text{C}})e^{-k_{\text{C}}x}\sin(k_{\text{C}}x - \omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_{\text{C}}^2)e^{-k_{\text{C}}x}\cos(k_{\text{C}}x - \omega t) + E_{\max}(+k_{\text{C}}^2)e^{-k_{\text{C}}x}\sin(k_{\text{C}}x - \omega t)$$
$$+ E_{\max}(+k_{\text{C}}^2)e^{-k_{\text{C}}x}\sin(k_{\text{C}}x - \omega t) + E_{\max}(-k_{\text{C}}^2)e^{-k_{\text{C}}x}\cos(k_{\text{C}}x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = -2E_{\max}k_{\text{C}}^2 e^{-k_{\text{C}}x}\cos(k_{\text{C}}x - \omega t).\ \frac{\partial E_y}{\partial t} = -E_{\max}e^{-k_{\text{C}}x}\omega\sin(k_{\text{C}}x - \omega t).$$

Setting $\dfrac{\partial^2 E_y}{\partial x^2} = \dfrac{\mu\partial E_y}{\rho\partial t}$ gives $2E_{\max}k_{\text{C}}^2 e^{-k_{\text{C}}x}\sin(k_{\text{C}}x - \omega t) = \mu/pE_{\max}e^{-k_{\text{C}}x}\omega\sin(k_{\text{C}}x - \omega t)$. This will only be true if $\dfrac{2k_{\text{C}}^2}{\omega} = \dfrac{\mu}{\rho}$, or $k_{\text{C}} = \sqrt{\dfrac{\omega\mu}{2\rho}}$.

(b) The energy in the wave is dissipated by the i^2R heating of the conductor.

(c) $E_y = \dfrac{E_{y0}}{e} \Rightarrow k_{\text{C}}x = 1,\ x = \dfrac{1}{k_{\text{C}}} = \sqrt{\dfrac{2\rho}{\omega\mu}} = \sqrt{\dfrac{2(1.72\times10^{-8}\ \Omega\cdot\text{m})}{2\pi(1.0\times10^6\ \text{Hz})\mu_0}} = 6.60\times10^{-5}\,\text{m}.$

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

32.57. **IDENTIFY:** The orbiting particle has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}$ kg and a proton has mass $m_p = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\left[\dfrac{q^2a^2}{6\pi\epsilon_0 c^3}\right] = \dfrac{C^2(m/s^2)^2}{(C^2/N\cdot m^2)(m/s)^3} = \dfrac{N\cdot m}{s} = \dfrac{J}{s} = W = \left[\dfrac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(6.00\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}{(1.67\times10^{-27}\ \text{kg})(0.75\ \text{m})} = 1.53\times10^{15}\ \text{m/s}^2$. The rate at which it emits energy because of its acceleration is

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.6\times10^{-19}\ \text{C})^2(1.53\times10^{15}\ \text{m/s}^2)^2}{6\pi\epsilon_0(3.0\times10^8\ \text{m/s})^3} = 1.33\times10^{-23}\ \text{J/s} = 8.32\times10^{-5}\ \text{eV/s}$.

Therefore, the fraction of its energy that it radiates every second is

$\dfrac{(dE/dt)(1\ \text{s})}{E} = \dfrac{8.32\times10^{-5}\ \text{eV}}{6.00\times10^6\ \text{eV}} = 1.39\times10^{-11}$.

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses: $E_e = E_p\dfrac{m_e}{m_p} = (6.00\times10^6\ \text{eV})\dfrac{(9.11\times10^{-31}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg})} = 3273\ \text{eV}$. Therefore, the fraction of its energy that it radiates every second is $\dfrac{(dE/dt)(1\ \text{s})}{E} = \dfrac{8.32\times10^{-5}\ \text{eV}}{3273\ \text{eV}} = 2.54\times10^{-8}$.

EVALUATE: The proton has speed $v = \sqrt{\dfrac{2E}{m_p}} = \sqrt{\dfrac{2(6.0\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.67\times10^{-27}\ \text{kg}}} = 3.39\times10^7\ \text{m/s}$. The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.58 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.58. **IDENTIFY:** The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\ \text{eV} = 1.60\times10^{-19}$ C. An electron has $|q| = e = 1.60\times10^{-19}$ C.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(13.6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(9.11\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})} = 9.03\times10^{22}\ \text{m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.57:

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.60\times10^{-19}\ \text{C})^2(9.03\times10^{22}\ \text{m/s}^2)^2}{6\pi\epsilon_0(3.00\times10^8\ \text{m/s})^3} = 4.64\times10^{-8}\ \text{J/s} = 2.89\times10^{11}\ \text{eV/s}$. This large value of $\dfrac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.57 must not apply to electrons in atoms.